

# Polyakov loop and Polyakov loop correlators in lattice QCD

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in collaboration with

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(**TUMQCD** collaboration)

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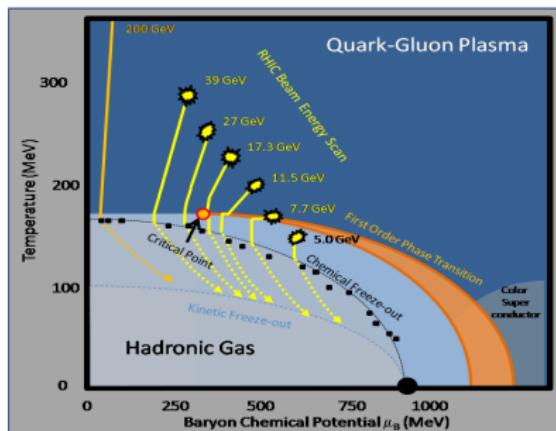


Riken BNL Research Center Workshop,  
Brookhaven National Lab, 02/13/2017

**PRD 93 114502 (2016); MPL A31** no.35, 1630040 (2016); arXiv:1601:08001

## Quark-Gluon-Plasma - the high-temperature phase of QCD

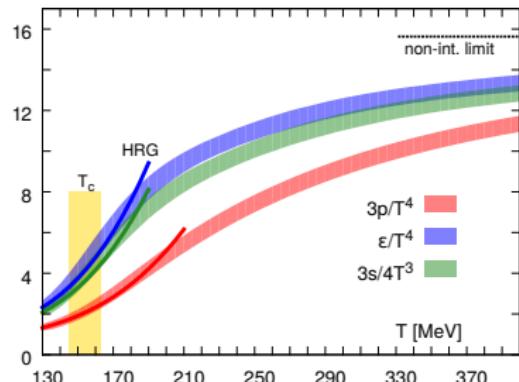
### QCD Phase diagram



(source: RIKEN BNL)

- Smooth crossover region
- Accidental symmetries are broken/restored in crossover.**

### QCD Equation of state



A. Bazavov et al., PRD 90 094503 (2014) [HotQCD]

- Increase of particle number –** HRG too low for  $T > 150$  MeV!
- $\mu > 0$  also available by now

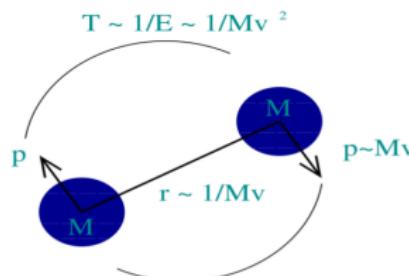
A. Bazavov et al., 1701.04325 (2017) [HotQCD]  
J. Günther et al., 1607.02493 (2016) [BW coll.]

## Color screening in a high temperature quark-gluon-plasma

- Introduction & Overview
- Effective field theories for heavy quarks
- Polyakov loops – color screening for a single quark
- Polyakov loop correlators – color screening for a quark-antiquark pair
- Summary

## Heavy quarkonium – the positronium of QCD

Upsilon ( $\Upsilon$ ) and Psi ( $\psi$ ) states are typical examples of quarkonia.



(figure by A. Vairo)

- Quarkonia are **non-relativistic bound states** of a **heavy** quark  $Q = \{b, c\}$  and a heavy anti-quark  $\bar{Q} = \{\bar{b}, \bar{c}\}$ .
- Hierarchy of scales** due to slow motion of quarks:  $M \gg Mv \gg Mv^2$
- Systematic expansion** in the quark mass is possible ( $\frac{\Lambda_{\text{QCD}}}{M} \ll 1$ ).

## Effective field theories for non-relativistic heavy quarkonia

### Potential Non-relativistic QCD

- Integrate out  $M \gg \mu \rightarrow \text{NRQCD}$  and then  $Mv \gg \mu_{\text{us}} \rightarrow \text{pNRQCD}$
- i.e. a **multipole expansion** in the *relative coordinate*  $r \sim \frac{1}{Mv} \sim \frac{1}{\Lambda_{\text{qcd}}}$ .
- *Color-singlet* and *color-octet fields* are the degrees of freedom.
- *Wilson coefficients* are **non-local**, depend on *relative coordinate*  $r$ .
- *Wilson coefficients* are **potentials**.

N. Brambilla et al., RMP77 1423 (2005)

### NRQCD at finite temperature

- Temperature  $T$  is parameter.
- **Thermal scales:**  $T, gT, g^2 T$
- $\frac{1}{r} \sim \Lambda_{\text{qcd}}$  vs thermal scales
- *Debye screening:*  $m_D \sim gT$ ,  $g = \sqrt{4\pi\alpha_s} \sim 1$  for  $T \sim 1 \text{ GeV}$
- *Landau damping* ( $E_b < m_D$ )
- *Gluodissociation* ( $E_b > m_D$ )
- **Potentials** become *complex*.

N. Brambilla et al., PRD78 014017 (2008)

M. Laine et al., JHEP 0703 054 (2007)

## Static potential at finite temperature

- Melting of quarkonia is controlled by the **screened, complex static potential**  $V_S(r, T)$ , which has been calculated at next-to-leading order:

$$V_S(r, T) = -C_F \alpha_s \left[ \frac{e^{-rm_D}}{r} + m_D + iT - \frac{2iT}{rm_D} \int_0^\infty dx \frac{\sin(rm_D x)}{(x^2+1)^2} \right] + \mathcal{O}(g^4).$$

M. Laine et al., JHEP 0703 054 (2007)

- $\text{Im } V_S \gg \text{Re } V_S \Rightarrow \text{color screening}$  not the mechanism behind melting.

P. Petreczky et al., NPA 855 125 (2011)

- No non-perturbative determination of  $V_S(r, T > 0)$  with a controlled error budget to date – *real-time properties such as complex potentials* at  $T > 0$  from *imaginary-time simulations* are *extremely difficult* at best.  
 ⇒ obtain constraints for *complex quantities* from purely *real quantities*.
- Singlet free energy** and real part of the **potential** appear to be related:

$$F_S(r, T) = \text{Re } V_S(r, T) + \mathcal{O}(g^4) = -C_F \alpha_s \left[ \frac{e^{-rm_D}}{r} + m_D \right] + \mathcal{O}(g^4).$$

N. Brambilla et al., PRD 82 (2010)

- Non-perturbatively:  $F_S \simeq \text{Re } V_S$ ? same  $m_D$  in  $F_S$ ,  $\text{Re } V_S$  and  $\text{Im } V_S$ ?

## Lattice gauge theory at finite temperature

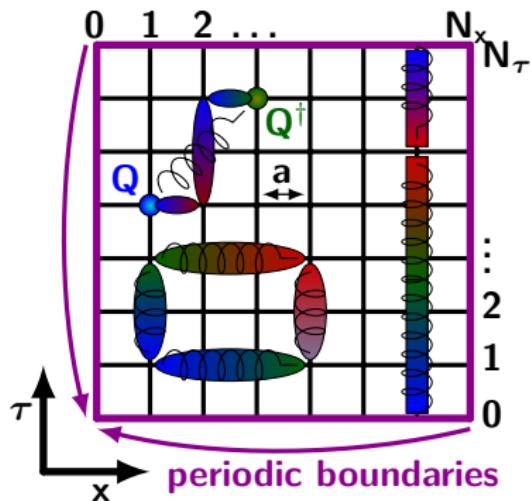
### Lattice QCD at finite temperature

- A *finite imaginary time direction* acts as an **inverse temperature**:

$$aN_\tau = \frac{1}{T}$$

- Correlations wrapping around the temporal lattice boundary* most sensitive to in-medium effects.
- The **continuum limit** ( $a \rightarrow 0$ ) at **fixed temperature  $T$**  is reached via concurrent modification of  $a$  and  $N_\tau$ : continuum at  $N_\tau \rightarrow \infty$ .

### Gauge-invariant operators on a euclidean space-time grid



## Polyakov loops and free energies of static quark states

- The *Polyakov loop*  $L$  is the gauge-invariant expectation value of the traced propagator of a static quark ( $P$ ) and related to its **free energy**:  

$$L(T) = \langle P \rangle_T = \langle \text{Tr } S_Q(x, x) \rangle_T = e^{-F_Q^b/T}$$
.  $L$  needs renormalization.

A. M. Polyakov, PL 72B (1978); L. McLerran, B. Svetitsky, PRD 24 (1981)

- The *Polyakov loop correlator* is related to *singlet* & *octet free energies*

$$C_P(r, T) = e^{-F_{Q\bar{Q}}^b(r, T)} = \frac{1}{9}e^{-F_S^b/T} + \frac{8}{9}e^{-F_A^b/T} = \frac{1}{9}C_S(r, T) + \frac{8}{9}C_A(r, T).$$

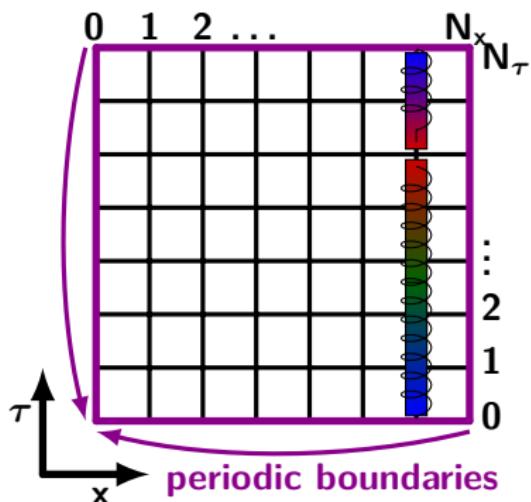
S. Nadkarni, PRD 33, 34 (1986)

- The *Polyakov loop correlator* is related to the **potentials** of pNRQCD

$$C_P(r, T) = e^{-F_{Q\bar{Q}}^b(r, T)} = \frac{1}{9}e^{-V_S^b/T} + \frac{8}{9}L_A^b e^{-V_A^b/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$

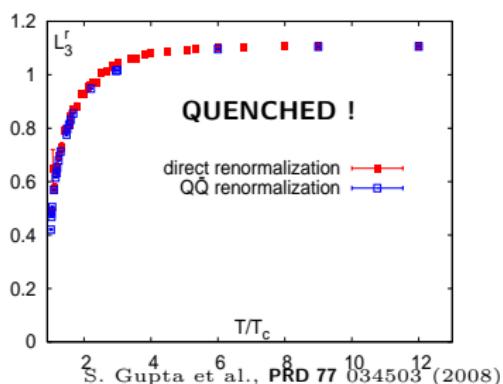
N. Brambilla et al., PRD 82 (2010)

## Color screening for a single static quark



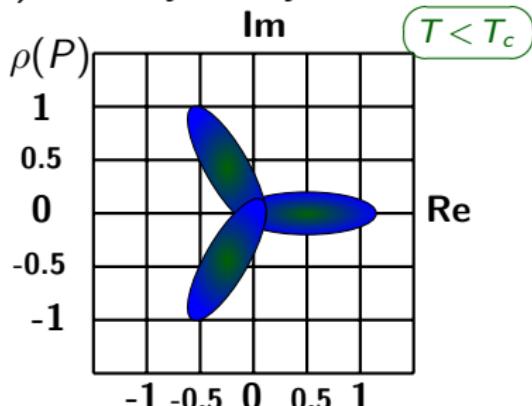
## The Polyakov loop as an order parameter

$L^r$  in SU(3) pure gauge theory



The Polyakov loop is an order parameter in pure gauge theory due to breaking Z(3) center symmetry.

Z(3) center symmetry as a cartoon

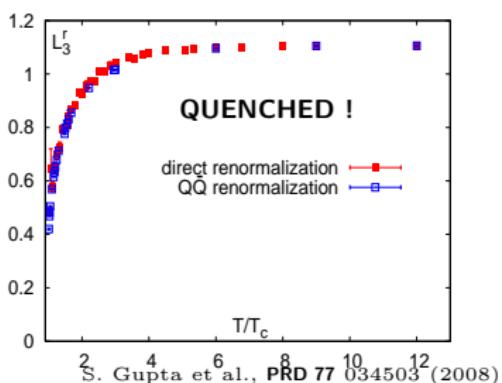


$$L = 0 \Leftrightarrow F_Q = \infty$$

Confinement in pure gauge theory

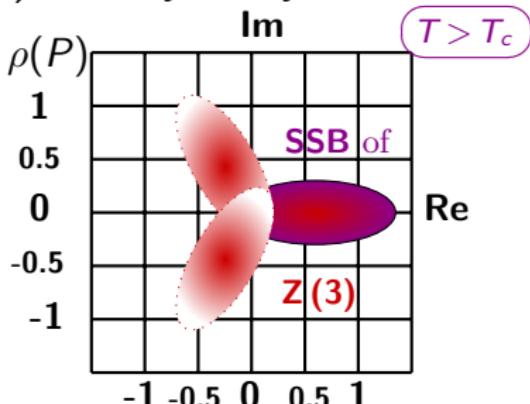
## The Polyakov loop as an order parameter

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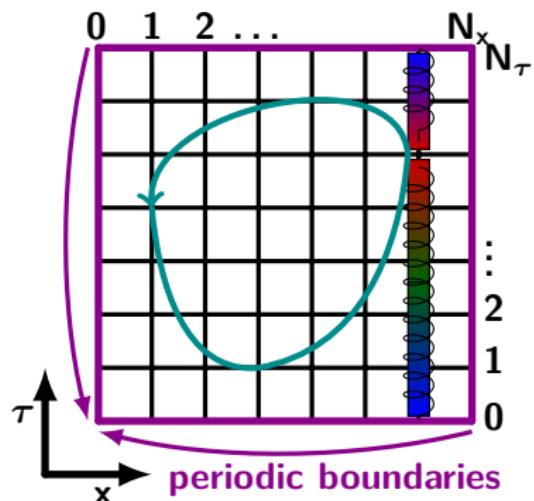
Z(3) center symmetry as a cartoon



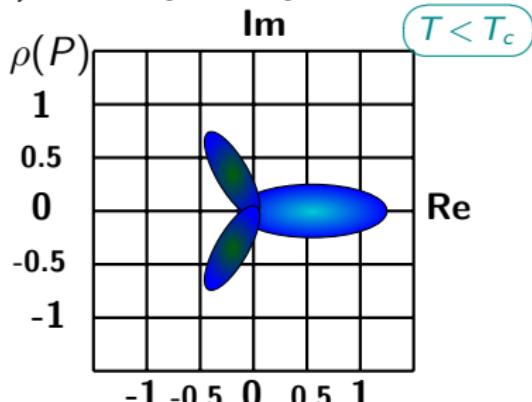
$L > 0 \Leftrightarrow F_Q < \infty$  (color screening)  
Deconfinement in pure gauge theory

## The Polyakov loop as an order parameter

Polyakov loop with sea quarks

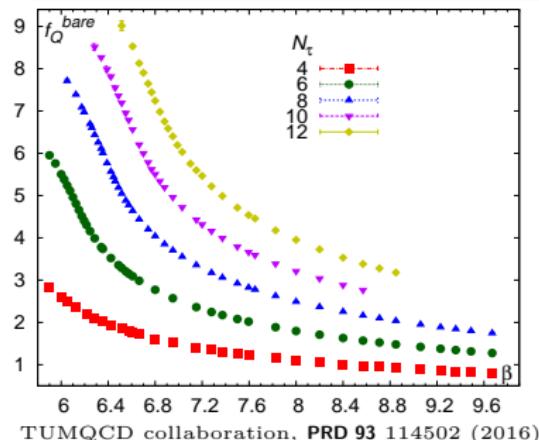


$Z(3)$  center symmetry as a cartoon

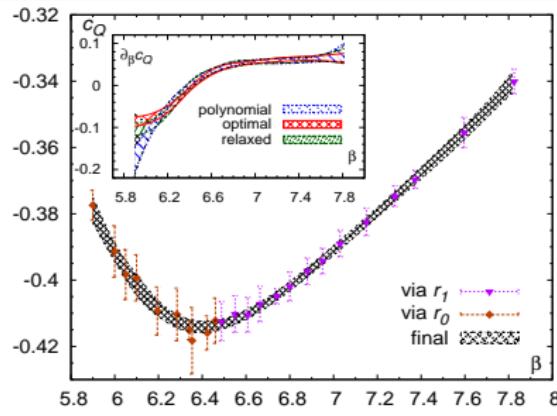


$L > 0 \Leftrightarrow F_Q < \infty$  (string breaking)  
 $F_Q \simeq \sum_i E_i$  due to static hadrons

## Bare Polyakov loop, bare free energy and renormalization



TUMQCD collaboration, PRD 93 114502 (2016)



$Q\bar{Q}$  procedure ( $T = 0$ )

The **free energy**  $f_Q^{\text{b}} \equiv \frac{F_Q^{\text{b}}}{T} = -\log L$  is **UV divergent**. Renormalization as  $L^{\text{r}} = L e^{-\frac{C_Q}{T}}$   $\leftrightarrow$   $f_Q = f_Q^{\text{b}} + \frac{C_Q}{T}$  with  $C_Q = \frac{c}{a} + b + \mathcal{O}(a^2)$  leads to a **scheme dependence**:  $b + \mathcal{O}(a^2)$

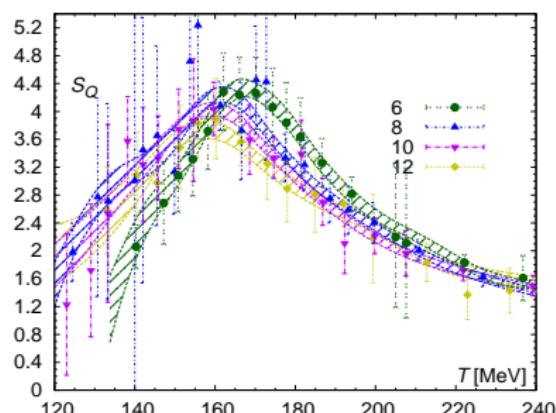
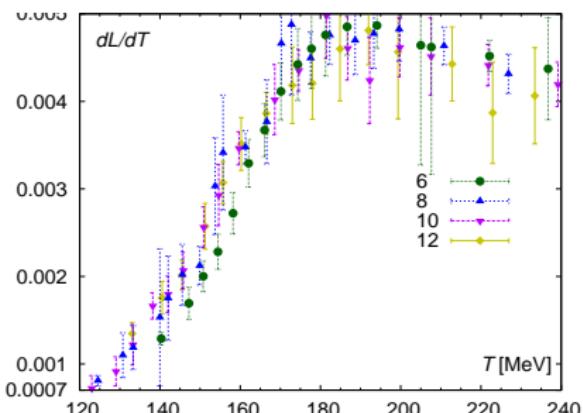
Fix  $V_S(r_i) = V_S^{\text{b}}(r_i) + 2C_Q(\beta) = \frac{V_i}{r_i}$  for  $V'_S(r_i) = r_i^2 C_i$ , where  $\{C_i, r_i\}$  are known from quark models.

**Large  $T = 0$  lattices are needed!**

A. Bazavov et al., PRD 85 054503 (2012) [HotQCD]

A. Bazavov et al., PRD 90 094503 (2014) [HotQCD]

## Critical behavior in renormalized Polyakov loop and free energy



**Temperature derivative of  $L$**

$\frac{dL}{dT}$  peaks at  $T \sim 190$  MeV

$\frac{dL}{dT}$  is explicitly **scheme dependent**,

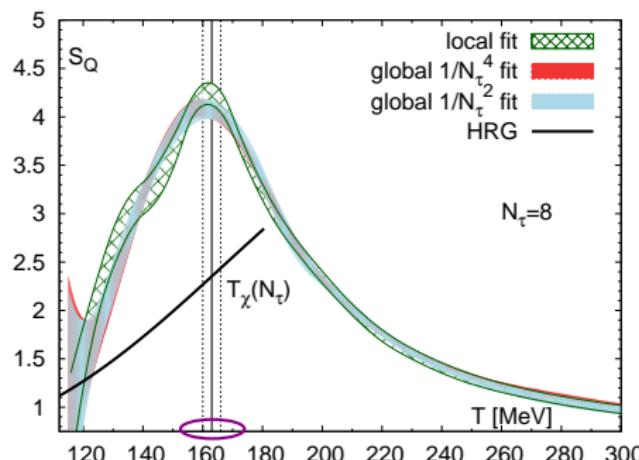
**Temperature derivative of  $F_Q$**

$S_Q = -\frac{dF_Q}{dT}$  peaks at  $T \sim 160$  MeV

though  $S_Q$  is a **measurable quantity**.

JHW, MPL A31 no.35, 1630040 (2016)

## $T_c$ from chiral observables vs the peak of the entropy

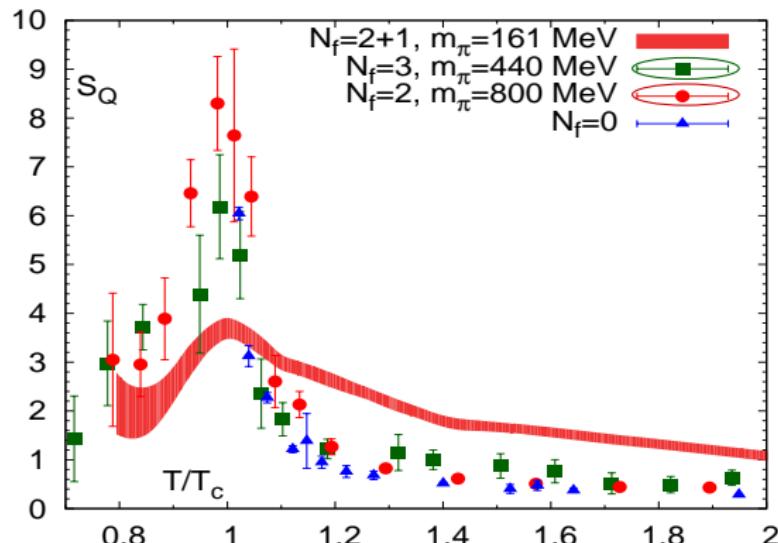


- $T_\chi$  defined via  $O(2)$  scaling of  $\chi_{m,l}$  ( $O(4)$ : 1–3.5 MeV lower  $T_\chi$ )

A. Bazavov et al., PRD 85 054503 (2012) [HotQCD]

- $T_S(N_t) \simeq T_\chi(N_t)$  for any  $N_t$  despite *different cutoff effects* suggests a **close connection of chiral symmetry and deconfinement**.
- *Hadron resonance gas* (HRG) limited to only below  $T \sim 125$  MeV.

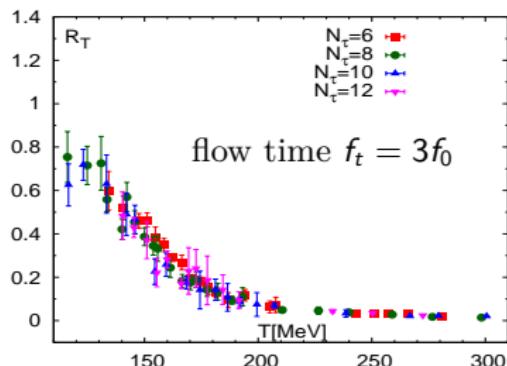
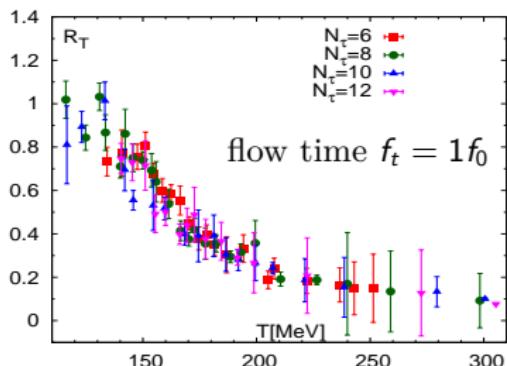
## Critical behavior of the entropy



O. Kaczmarek, F. Zantow,  
[hep-lat/0506019 \(2005\)](#)  
 P. Petreczky, K. Petrov  
[PRD 70 054503 \(2004\)](#)

- The **peak decreases for lower quark masses** and for finer lattices.  
 → interpret critical behavior as **melting of the static-light mesons**.
- The entropy peaks at  $T_S = 153^{+6.5}_{-5} \text{ MeV}$  in the continuum limit.

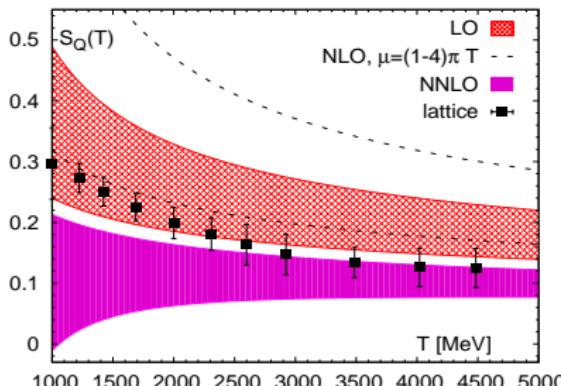
## Critical behavior of Polyakov loop susceptibilities



- Ratios of **longitudinal** and **transverse** Polyakov loop susceptibilities:  

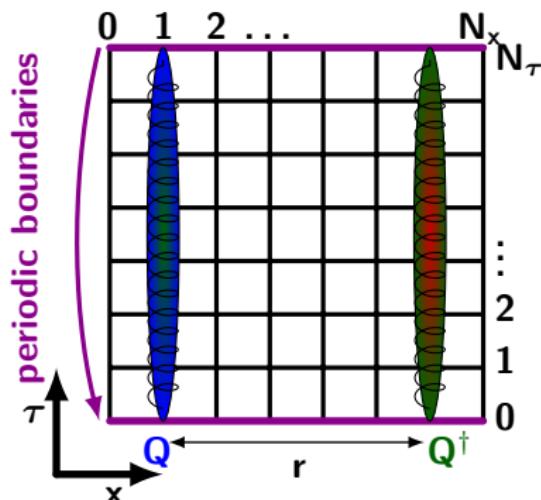
$$\frac{\chi_L}{VT^3} = [\langle \text{Re } P^2 \rangle - \langle \text{Re } P \rangle^2], \quad \frac{\chi_T}{VT^3} = \langle \text{Im } P^2 \rangle$$
P. Lo et al., PRD 88 014506 (2013)
- We use gradient flow for renormalization.  
M. Lüscher, JHEP 1008 071 (2010)
- $R_T = \chi_T / \chi_L$ : **crossover pattern** for  $f_t \geq f_0$ , exposes **critical behavior**.

## Confronting weak-coupling predictions at high temperatures

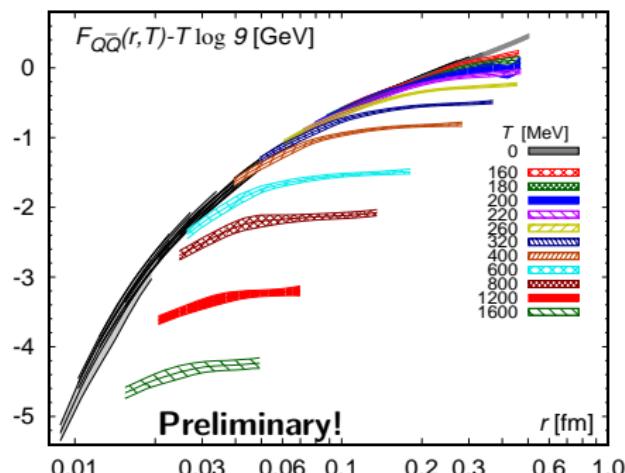


- Discretization effects are very mild for  $T > 500$  MeV.
  - We compare  $S_Q(T, 4)$  with a weak-coupling calculation for 3 flavors.
- M. Berwein et al., PRD 93 034010 (2016)
- For  $T \gtrsim 3$  GeV,  $S_Q(T, 4)$  agrees with NNLO, similar to pure SU(3).
  - Higher temperature than for quark number susceptibilities ( $T_{qns} \sim 300$  MeV) due to static Matsubara mode contribution to  $S_Q$ .

## Color screening for a static quark-antiquark pair



## Polyakov loop correlator and $Q\bar{Q}$ free energy

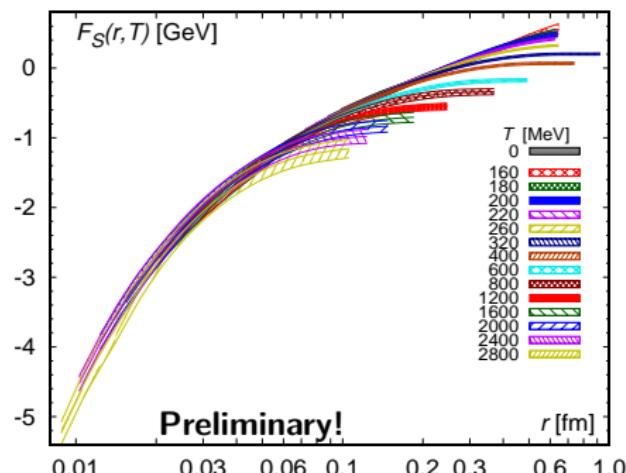


- Free energy of a  $Q\bar{Q}$  pair,  $F_{Q\bar{Q}}$ , is also called *color-averaged potential*:

$$C_P(r, T) = \langle P(0)P^\dagger(r) \rangle_T = e^{-\frac{F_{Q\bar{Q}}(r, T)}{T}} = \frac{1}{9}e^{-\frac{F_S(r, T)}{T}} + \frac{8}{9}e^{-\frac{F_A(r, T)}{T}}.$$

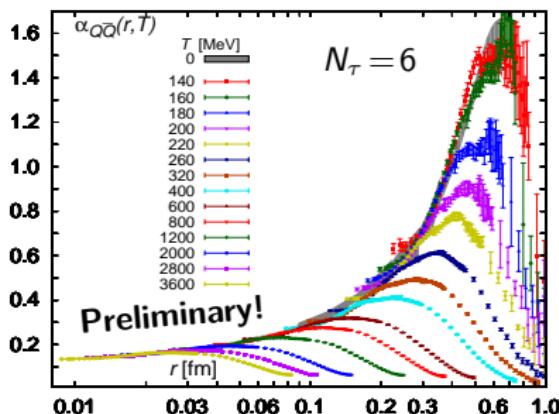
- $F_{Q\bar{Q}} - T \log 9$  is rather close to  $T=0$  static energy  $V_S$  up to  $rT \sim 0.15$ .

## Singlet free energy in Coulomb gauge



- **Singlet free energy:**  $C_S(r, T) = \frac{1}{3} \left\langle \sum_{a=1}^3 W_a(0) W_a^\dagger(r) \right\rangle_T = e^{-F_S(r, T)/T}$
- Wilson line correlator requires explicit **gauge fixing** (Coulomb gauge)
- $F_S$  is rather consistent with  **$T=0$  static energy**  $V_S(r)$  up to  $rT \sim 0.35$ .

## Effective coupling: confining and screening regimes

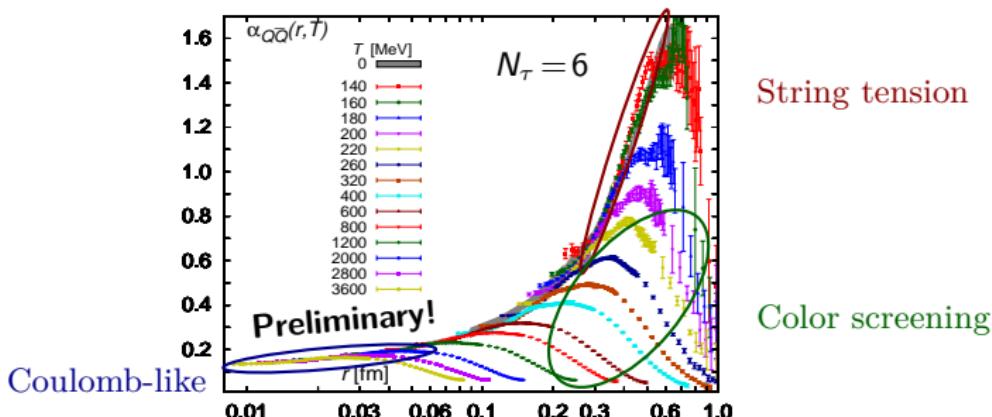


- **Effective coupling**  $\alpha_{Q\bar{Q}}(r, T)$  is a proxy for the **force** between  $Q$  and  $\bar{Q}$ .

$$\alpha_{Q\bar{Q}}(r, T) = \frac{r^2}{c_F} \frac{\partial E(r, T)}{\partial r}, \quad E = \{F_S(r, T), V_S(r)\}$$

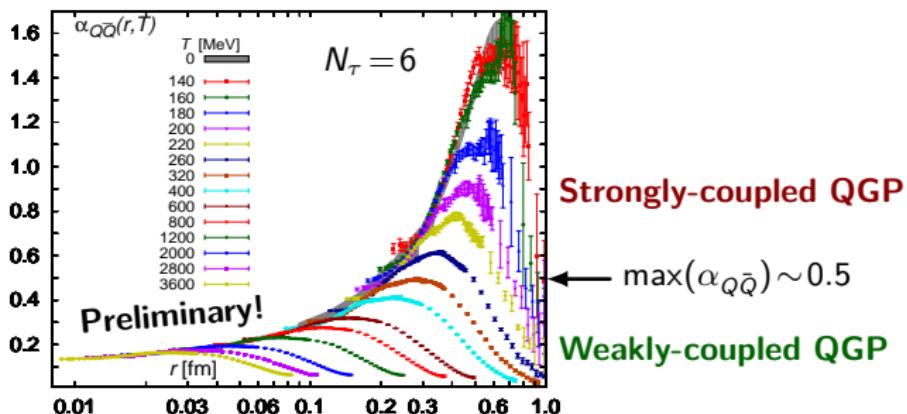
- $\alpha_{Q\bar{Q}}$  clearly distinguishes two different regimes at small and large  $r$ .

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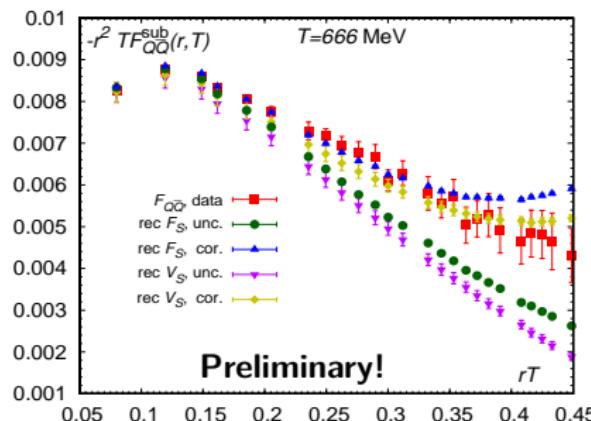


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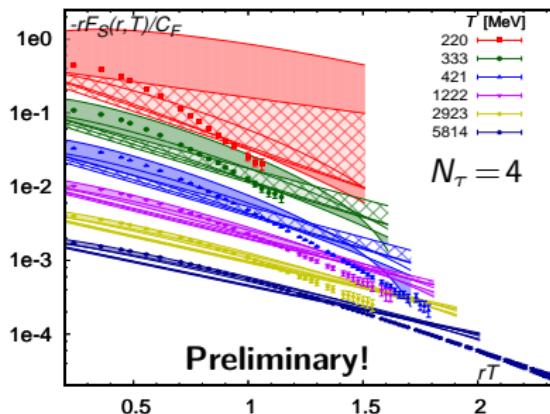
## Confronting weak-coupling predictions at short distances



- $pNRQCD$ :  $C_P$  is given in terms of **potentials**  $V_S$  and  $V_A$  at  $T=0$  and of the *adjoint Polyakov loop*  $L_A$  at  $T>0$  N. Brambilla et al., PRD 82 (2010)

$$C_P(r, T) = e^{-F_{Q\bar{Q}}(r, T)} = \frac{1}{9}e^{-V_S/T} + \frac{8}{9}L_A e^{-V_A/T} + \mathcal{O}(g^6) \text{ for } rT \ll 1.$$
- We reconstruct  $V_A$  from  $V_S$  and  $L_A$  from  $L$  via **Casimir scaling** and include the **Casimir scaling violation**:  $8V_A + V_S = 3\frac{\alpha_s^3}{r}[\frac{\pi^2}{4} - 3] + \mathcal{O}(\alpha_s^4)$ .

## Confronting weak-coupling predictions in the screening regime (I)

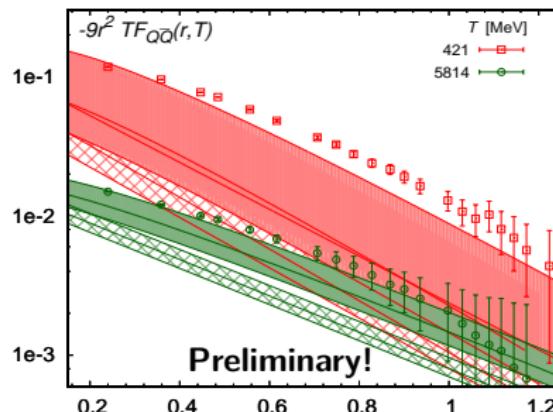


Hashed bands: LO  
Solid bands: NLO

Scale uncertainty  
 $\mu = (1-4)\pi T$   
 due to resummation

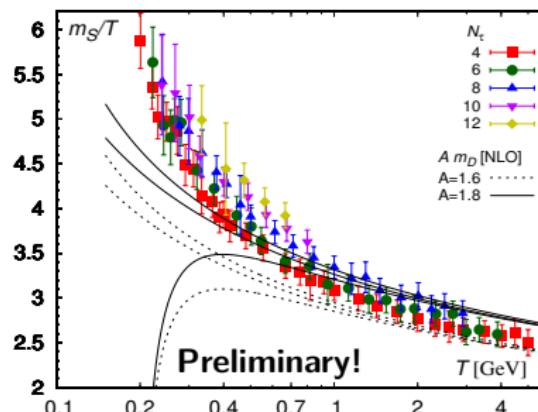
- $F_S(r, T) = -C_F \alpha_s \left[ \frac{e^{-rm_D}}{r} + m_D \right] + \mathcal{O}(g^4)$  was calculated in the electric screening regime at NLO by Laine et al. M. Laine et al., JHEP 0703 054 (2007)
- Lattice and NLO screening masses are similar at  $rT \sim 0.3$ , the singlet free energy still compatible up to  $rT \sim 0.8$  due to scale uncertainty.

## Confronting weak-coupling predictions in the screening regime (II)



- Leading order free energy:  $F_{Q\bar{Q}}(r, T) = -\frac{\alpha_s^2}{9} \frac{e^{-2rm_D}}{r^2} + C_F \alpha_s m_D$ .
- The **perturbation series of  $F_{Q\bar{Q}}$  breaks down** in the screening regime:  
NLO exceeds LO, **NNLO is non-perturbative!** S. Nadkarni, PRD 33 (1986)
- The NLO result is very close to the lattice data for  $rT \lesssim 0.4$ .

## Asymptotic singlet screening mass



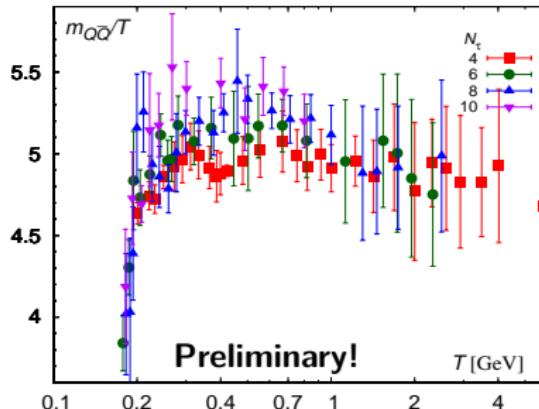
Triplets of lines:

Scale uncertainty  
 $\mu = (1-4)\pi T$   
 due to resummation

- Color screening becomes stronger for larger distances and any free energies must reach asymptotic screening behavior:  $F = -a \frac{e^{-rm}}{r} + c$ .
- The asymptotic **singlet screening mass  $m_S$**  exceeds the NLO Debye mass (electric mass in Electrostatic QCD). E. Braaten, A. Nieto, **PRD 53** (1996).
- Asymptotic and rescaled NLO masses share similar  $T$  dependence.

O. Kaczmarek, **PoS CPOD07** (2007).

## Asymptotic screening mass of $F_{Q\bar{Q}}$



- The screening mass  $m_{Q\bar{Q}}$  is already **at  $rT \sim 0.45$  asymptotic**.
- $\frac{m_{Q\bar{Q}}}{T}$  is **at most mildly temperature dependent** for  $T > 200$  MeV.
- $m_{Q\bar{Q}}$  is compatible with the **magnetic mass**  $m_M$  from smeared Polyakov loop correlators and with the ground state of massless  $N_f = 3$  EQCD.

S. Borsányi et al., JHEP 1504 138 (2015) [BW coll.]; A. Hart et al., NPB 586 (2000)

- We study color screening and deconfinement using the renormalized Polyakov loop and related observables.
- We see in the entropy  $S_Q = -\frac{dF_Q}{dT}$  and in the ratio of Polyakov susceptibilities  $R_T = \frac{\chi_T}{\chi_L}$  crossover behavior at  $T \sim T_c$ .
- We extract  $T_S = 153_{-5}^{+6.5}$  MeV from the entropy, in agreement with  $T_\chi = 160(6)$  MeV (chiral susceptibilities, O(2) scaling fits,  $\frac{m_l}{m_s} = \frac{1}{20}$ ).

$N_\tau$	$\infty$	12	10	8	6
$T_S$	$153_{-5}^{+6.5}$	157.5(6)	159(4.5)	162(4.5)	167.5(4.5)
$T_\chi$	160(6)	161(2)	[162(2)]*	164(2)	171(2)

- Weak-coupling behavior of the Polyakov loop sets in for  $T \sim 3$  GeV.

Color screening permits to precisely measure the onset of deconfinement.

- Continuum limit of static quark correlators in  $N_f = 2+1$  QCD up to  $T \sim 2.8$  GeV and down to  $r \sim 0.018$  fm.
- Static  $Q\bar{Q}$  correlators show **remnants of confinement**, and up to  $T \sim 300$  MeV QGP is strongly coupled.
- Onset of thermal effects is much stronger if **color-octet states** contribute.
- The free energy  $F_{qq}$  is given in terms of  **$T=0$  potentials and the adjoint Polyakov loop at  $T>0$**  in line with weakly-coupled  $pNRQCD$ .
- We confirm **electric screening** in both  $F_{Q\bar{Q}}$  and  $F_S$  at  $rT \sim 0.25$ .
- The screening mass of  $m_{Q\bar{Q}}$  is consistent with **EQCD predictions for the lowest scalar glueball** and has a trivial temperature dependence.

Color screening plays essentially no role in sequential melting, which is a consequence of quarkonium dissociation.